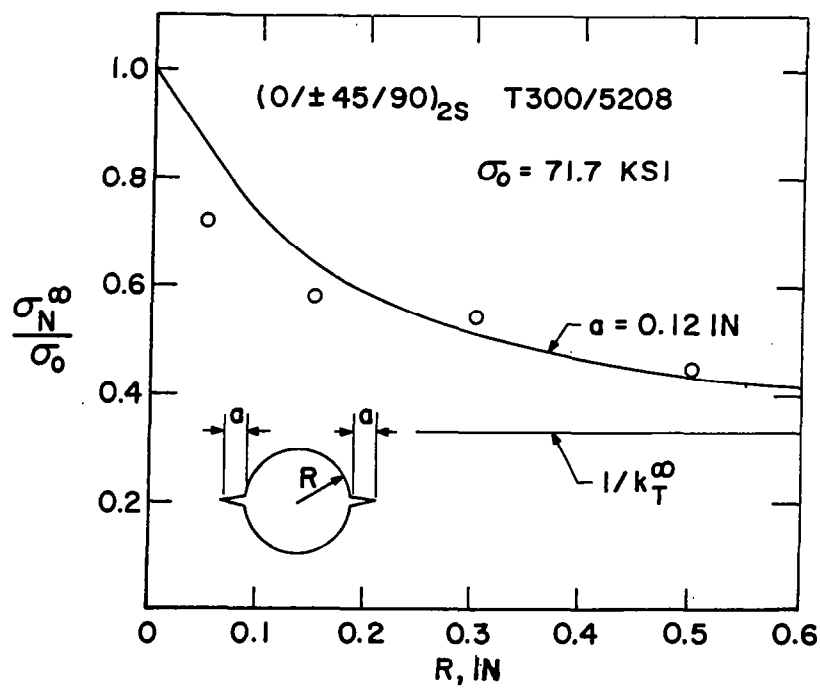
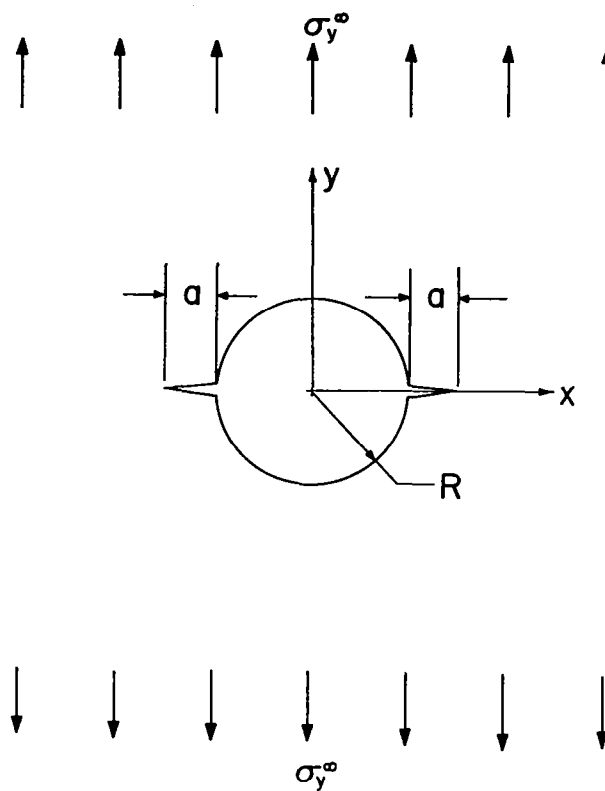


**NOTCH STRENGTH OF COMPOSITES**

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## STATISTICAL BEHAVIOR OF COMPOSITES

### WEIBULL STATISTICAL STRENGTH THEORY

$$R(S) = \exp \left[ -B(S) \right]$$

$B(S)$  = RISK - TO - BREAK

### TWO PARAMETER WEIBULL DISTRIBUTION

$$B(S) = \int_V \left( \frac{S}{S_0} \right)^\alpha dV$$

TENSION:  $B_t = V_t \left( \frac{S}{S_0} \right)^\alpha$

$$R(S) = \exp \left[ - \left( \frac{S}{S_t} \right)^\alpha \right]$$

$$S_t = \frac{S_0}{(V_t)^{1/\alpha}}$$

# BENDING

$$R(S) = \text{EXP} \left[ \left( -\frac{S}{S_b} \right)^\alpha \right]$$

$$\begin{aligned} \underline{4 \text{ PT:}} \quad B_b &= \frac{(\alpha + 2) V_b}{4(\alpha + 1)^2} \left( \frac{S}{S_0} \right)^\alpha \\ S_b &= S_0 \left[ \frac{4 (\alpha + 1)^2}{(\alpha + 2) V_b} \right]^{1/\alpha} \end{aligned}$$

$$\begin{aligned} \underline{3 \text{ PT:}} \quad B_b &= \frac{V_b}{2 (\alpha + 1)^2} \left( \frac{S}{S_0} \right)^\alpha \\ S_b &= S_0 \left[ \frac{2 (\alpha + 1)^2}{V_b} \right]^{1/\alpha} \end{aligned}$$

## BENDING (3 PT):

$$\frac{S_b}{S_t} = \left[ 2 (\alpha + 1)^2 \left( \frac{V_t}{V_b} \right) \right]^{1/\alpha} = \begin{cases} 1.52, \alpha = 15 \\ 1.33, \alpha = 25 \end{cases}$$

## BENDING (4 PT):

$$\frac{S_b}{S_t} = \left[ \frac{4 (\alpha + 1)^2}{(\alpha + 2)} \left( \frac{V_t}{V_b} \right) \right]^{1/\alpha} = \begin{cases} 1.31, \alpha = 15 \\ 1.20, \alpha = 25 \end{cases}$$

# THICKNESS EFFECTS

TENSION:

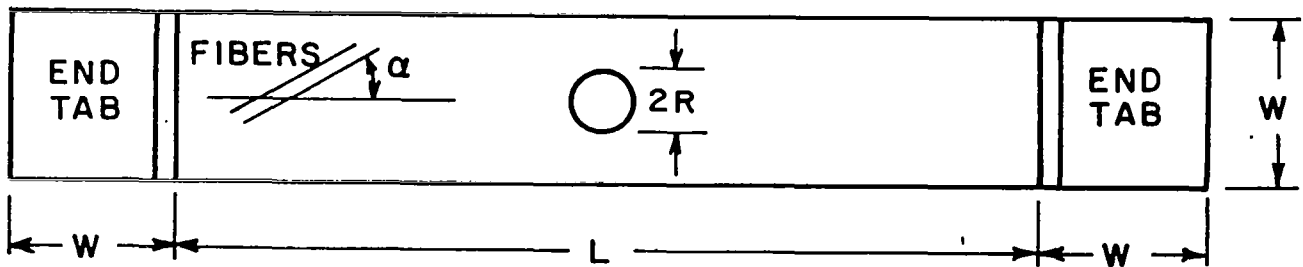
$$S_t = S_0 \left( \frac{1}{L_b h} \right)^{1/\alpha}$$

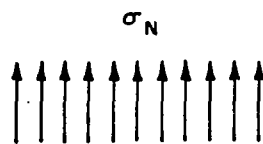
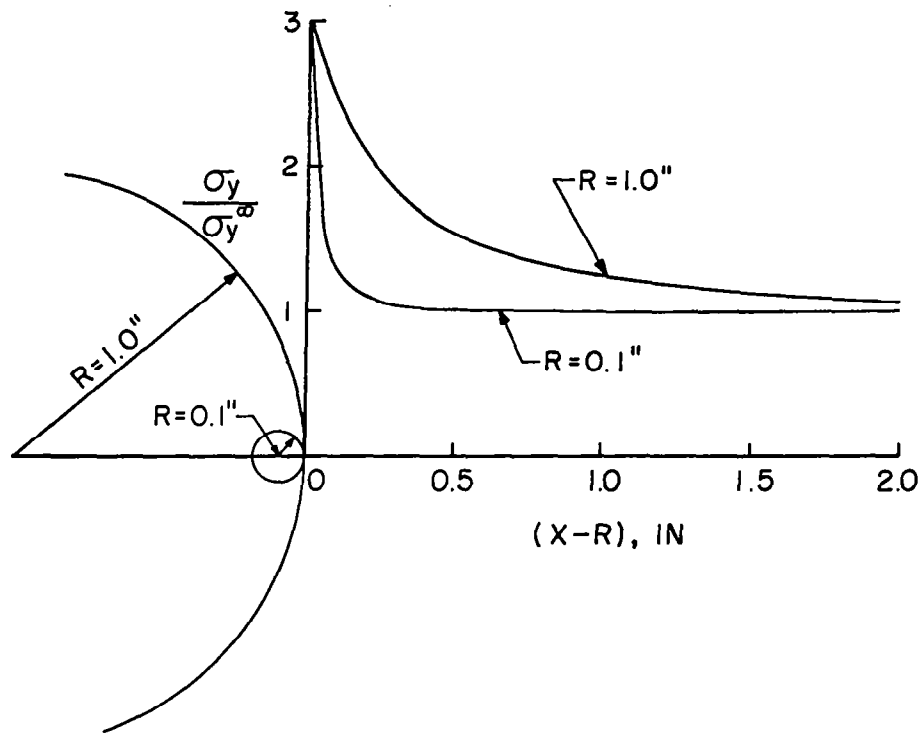
$$\frac{S_{t1}}{S_{t2}} = \left( \frac{h_2}{h_1} \right)^{1/\alpha}, \quad h_2 > h_1$$

BENDING:  
(3 PT)

$$S_b = \left[ S_0 \frac{(\alpha + 1)^2 h}{L_b h^2} \right]^{1/\alpha}$$

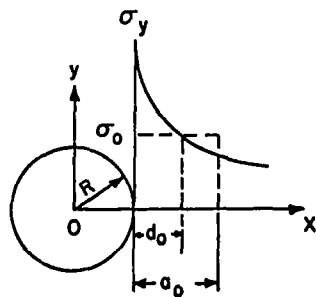
$$\frac{S_{b1}}{S_{b2}} = \left( \frac{h_2}{h_1} \right)^{2/\alpha}, \quad h_2 > h_1$$





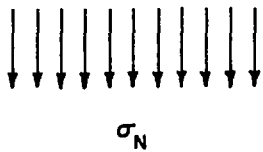
#### POINT STRESS CRITERION

$$\sigma_y (R+d_0, 0) = \sigma_0$$



#### AVERAGE STRESS CRITERION

$$\frac{1}{a_0} \int_R^{R+d_0} \sigma_y (x, 0) dx = \sigma_0$$



# CIRCULAR HOLE APPROXIMATE STRESSES

$$\sigma_y(x,0) = \frac{\sigma_y^\infty}{2} \left\{ 1 + \left(\frac{R}{X}\right)^2 + 3\left(\frac{R}{X}\right)^4 - (K_T - 3) \left[ 5\left(\frac{R}{X}\right)^6 - 7\left(\frac{R}{X}\right)^8 \right] \right\}$$

$X > R$

$$K_T^\infty = 1 + \sqrt{2 \left( \sqrt{\frac{\bar{E}_2}{\bar{E}_1}} - \nu_{12} \right) + \frac{\bar{E}_2}{\bar{G}_{12}}}$$

## CIRCULAR HOLE

### POINT STRESS CRITERION

$$\frac{\sigma_N^\infty}{\sigma_0} = \frac{2}{\left[ 2 + \xi_1^2 + 3\xi_1^4 - (K_T^\infty - 3)(5\xi_1^6 - 7\xi_1^8) \right]}$$

$$\xi_1 = \frac{R}{R + d_0}$$

### AVERAGE STRESS CRITERION

$$\frac{\sigma_N^\infty}{\sigma_0} = \frac{2(1 - \xi_2)}{\left[ 2 - \xi_2^2 - \xi_2^4 + (K_T^\infty - 3)(\xi_2^6 - \xi_2^8) \right]}$$

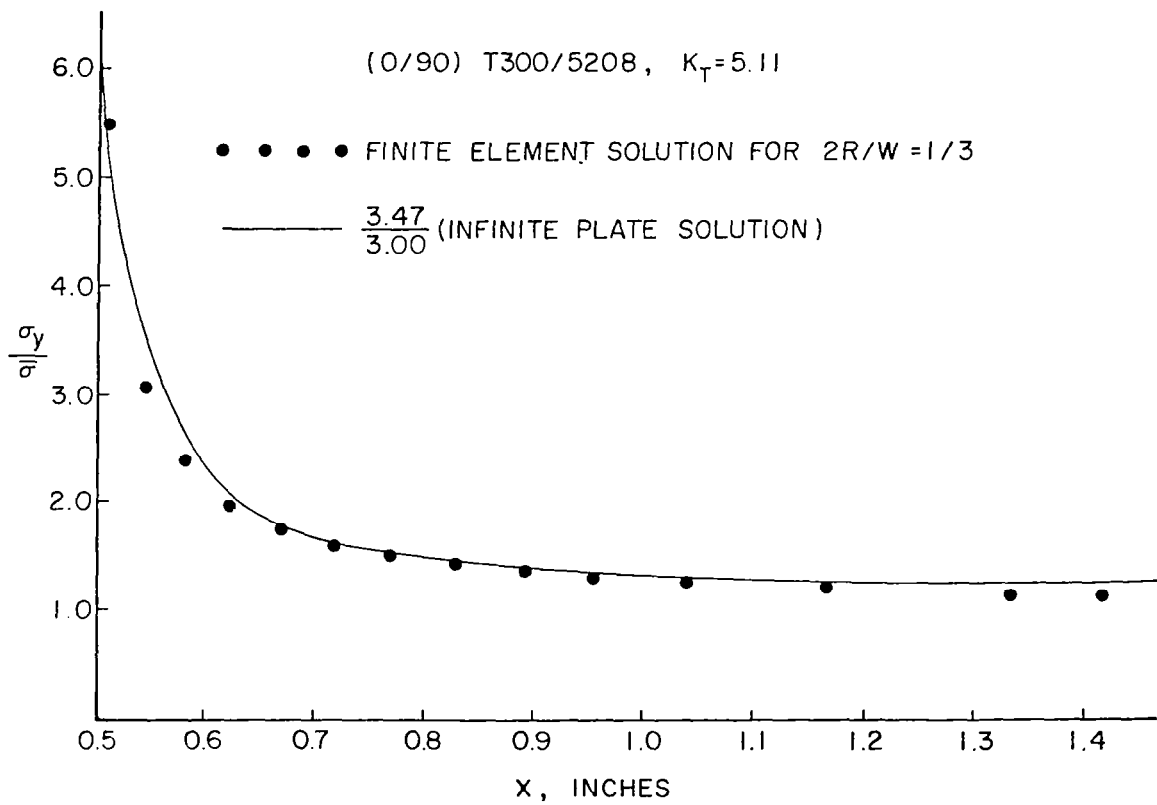
$$\xi_2 = \frac{R}{R + a_0}$$

# FINITE WIDTH CORRECTION -

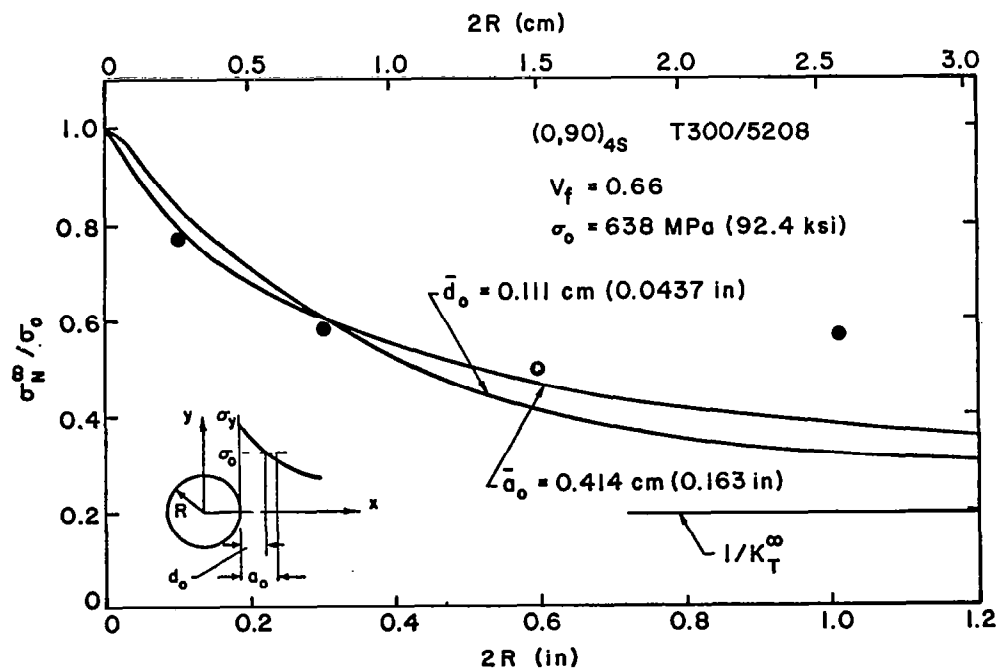
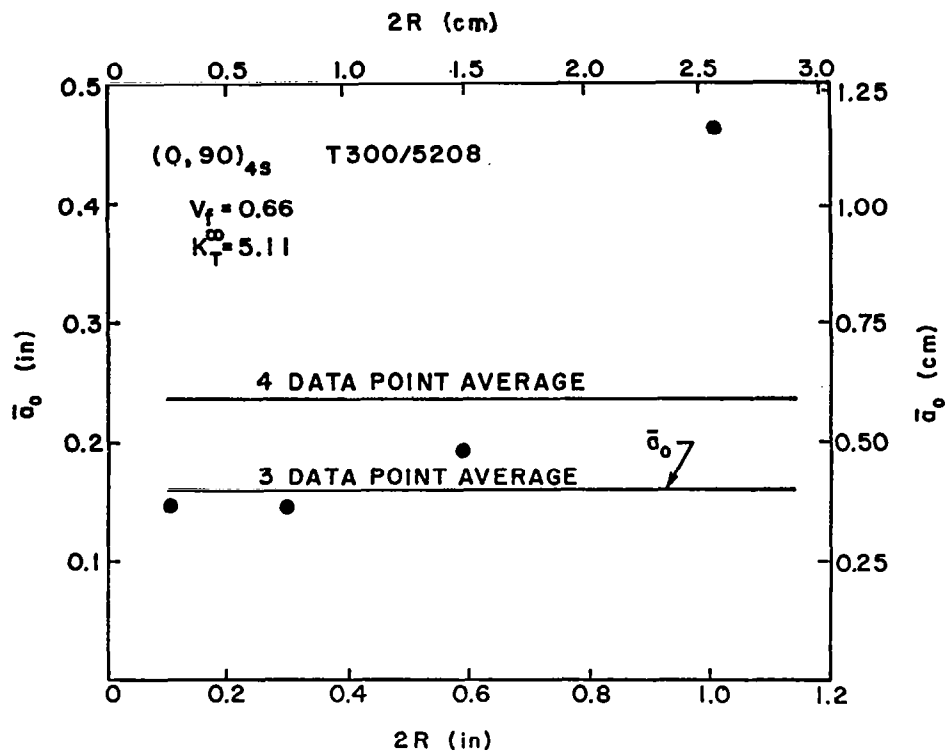
## CIRCULAR HOLE

$$\sigma_N^{\infty} = Y_3 (2R/W) \sigma_N$$

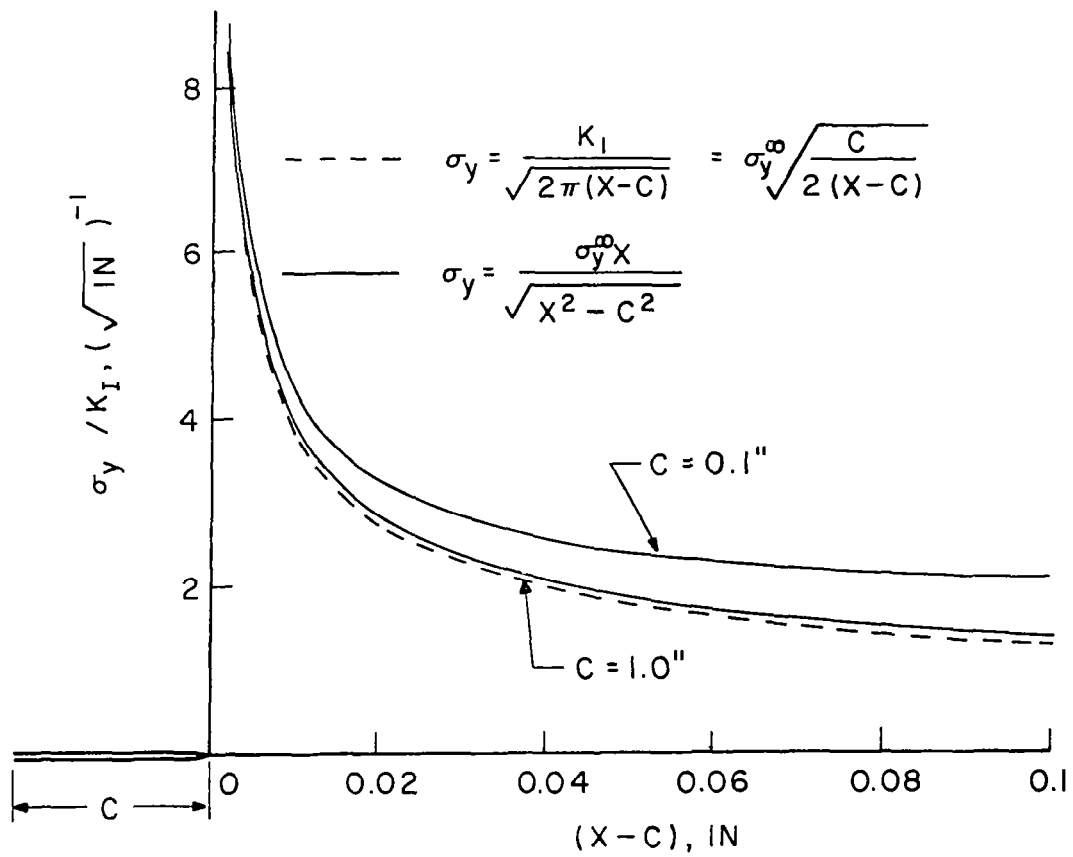
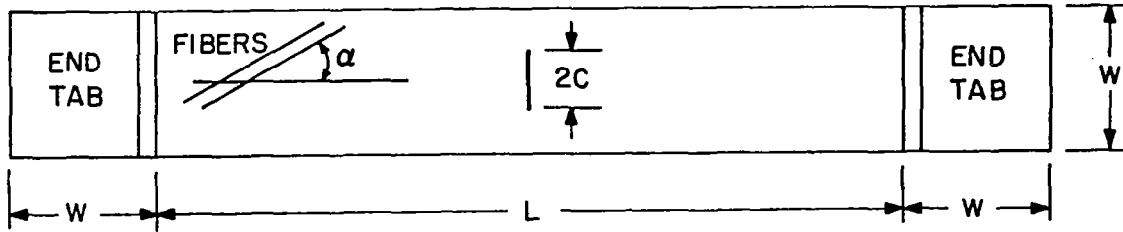
$$Y_3(2R/W) = \frac{2 + (1 - 2R/W)^3}{3(1 - 2R/W)}$$







# CENTER-CRACKED SPECIMEN



# CENTER CRACK

$$\sigma_y(X,0) = \frac{\sigma_y^\infty X}{\sqrt{X^2 - c^2}} = \frac{K_I X}{\sqrt{\pi c (X^2 - c^2)}} , X > c$$

## POINT STRESS CRITERION

$$\frac{\sigma_N^\infty}{\sigma_0} = \sqrt{1 - \xi_3^2} , \quad \xi_3 = \frac{c}{c + d_0}$$

## AVERAGE STRESS CRITERION

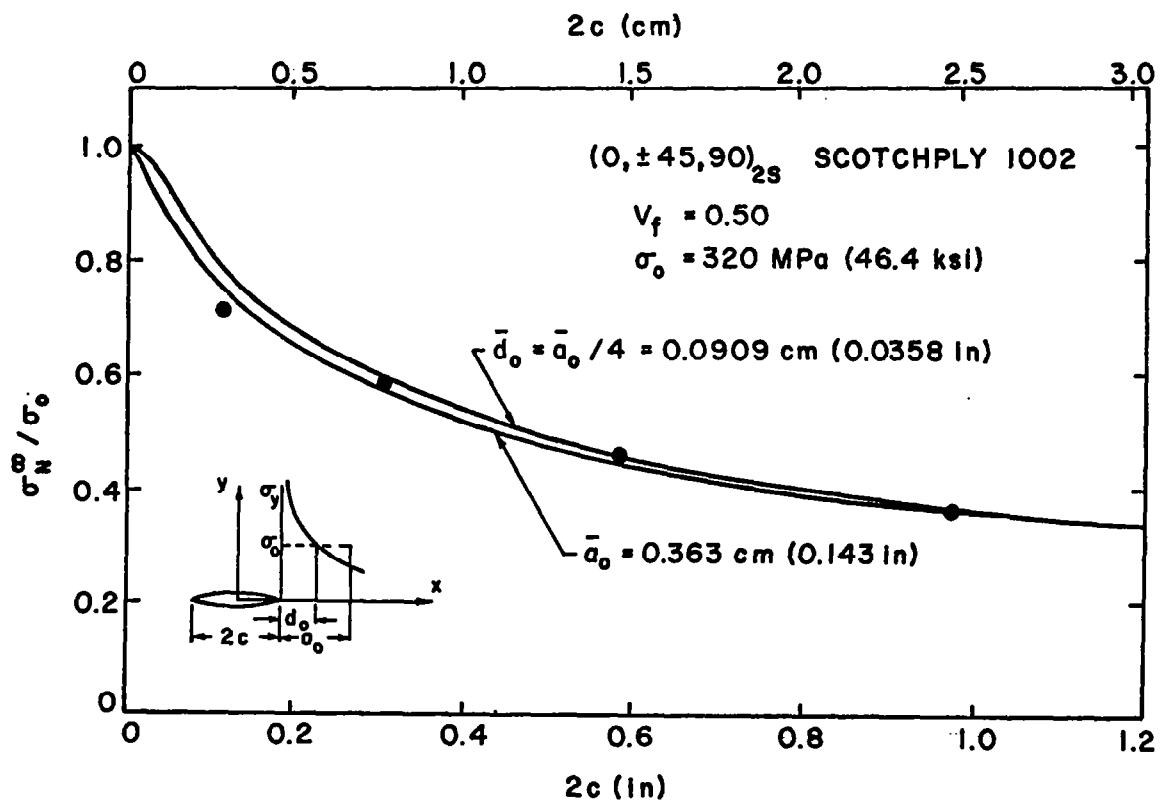
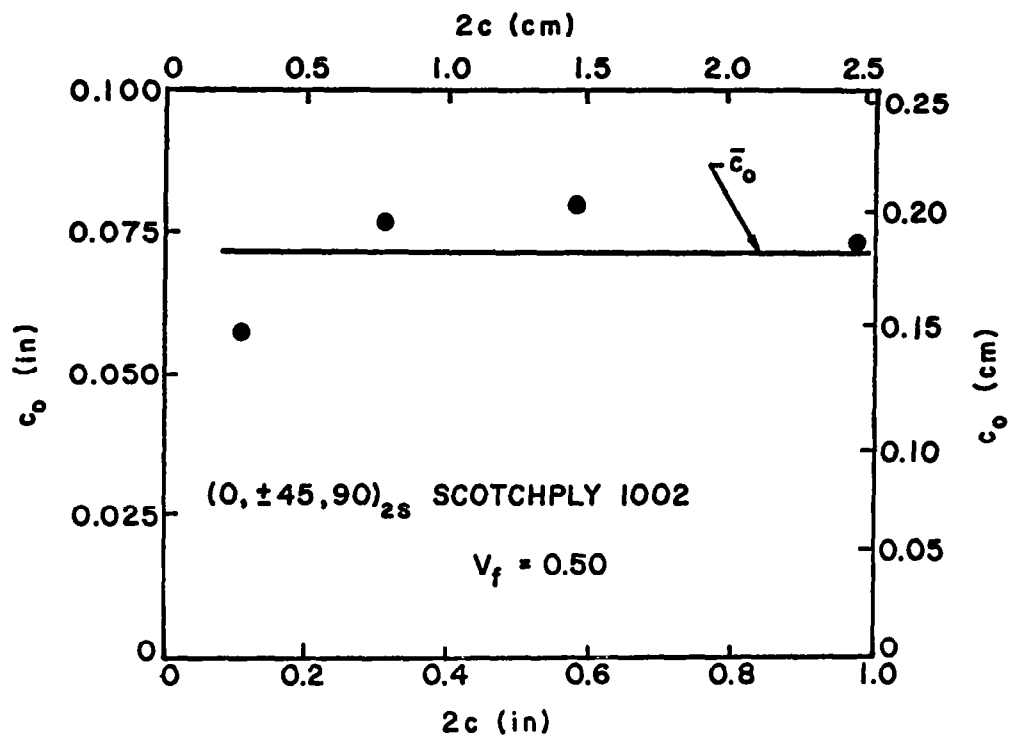
$$\frac{\sigma_N^\infty}{\sigma_0} = \sqrt{\frac{1 - \xi_4}{1 + \xi_4}} , \quad \xi_4 = \frac{c}{c + a_0}$$

## FINITE WIDTH CORRECTION - CENTER CRACK

$$\sigma_N^\infty = Y_1 (2c/w) \sigma_N$$

$$Y_1 (2c/w) = 1 + 0.128 (2c/w) - 0.288 (2c/w)^2 \\ + 1.52 (2c/w)^3$$

$$2c/w \leq 0.7, \text{ ERROR} < 0.5\%$$



# FRACTURE TOUGHNESS

## POINT STRESS CRITERION

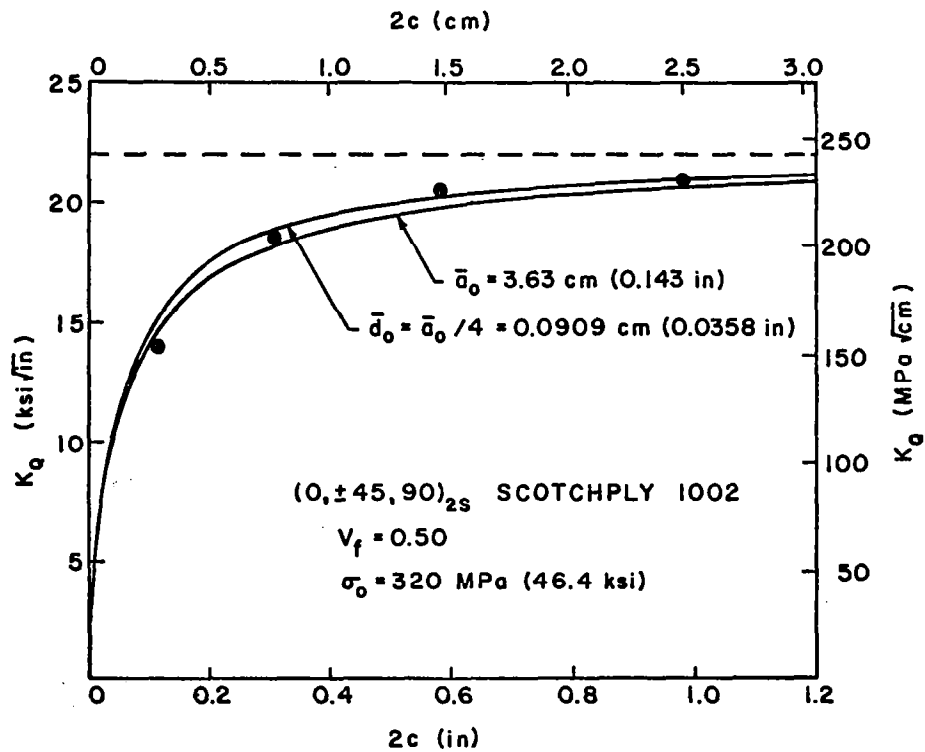
$$K_Q = \sigma_0 \sqrt{\pi c (1 - \xi_3^2)}$$

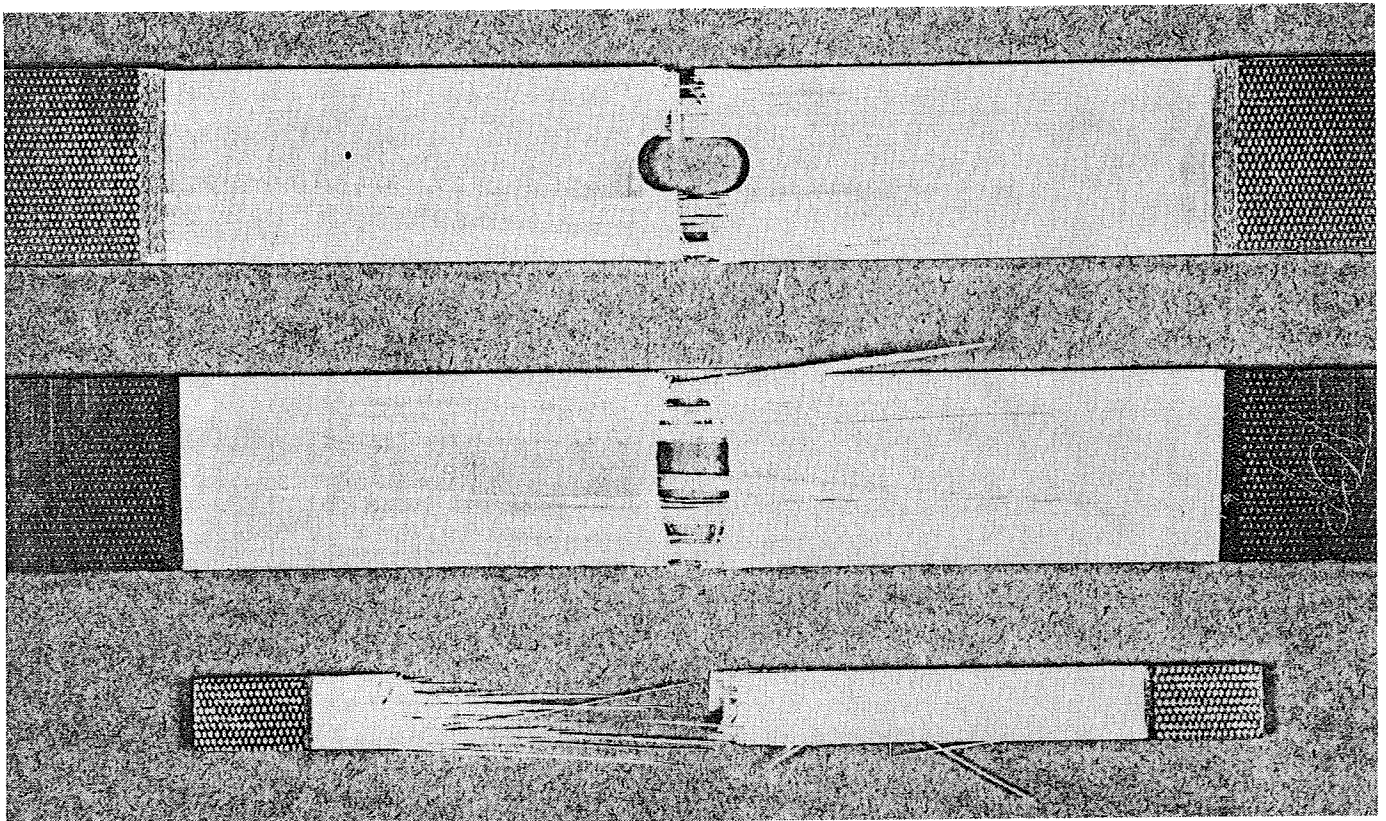
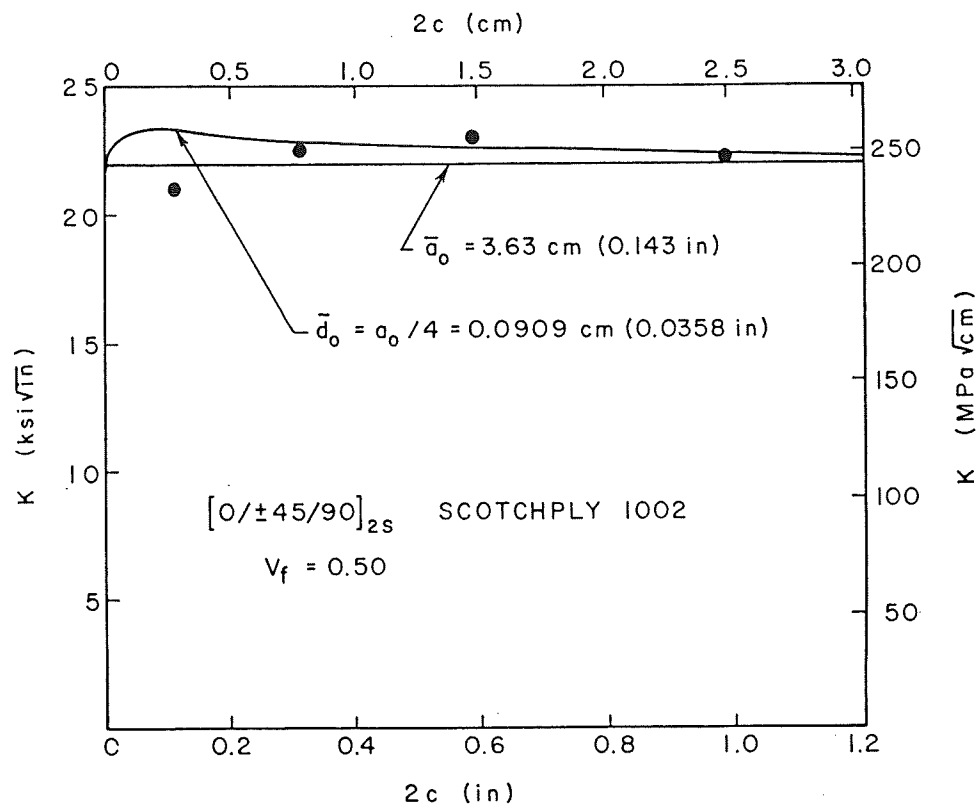
$$c \rightarrow \infty, K_Q \rightarrow \sigma_0 \sqrt{2\pi d_0}$$

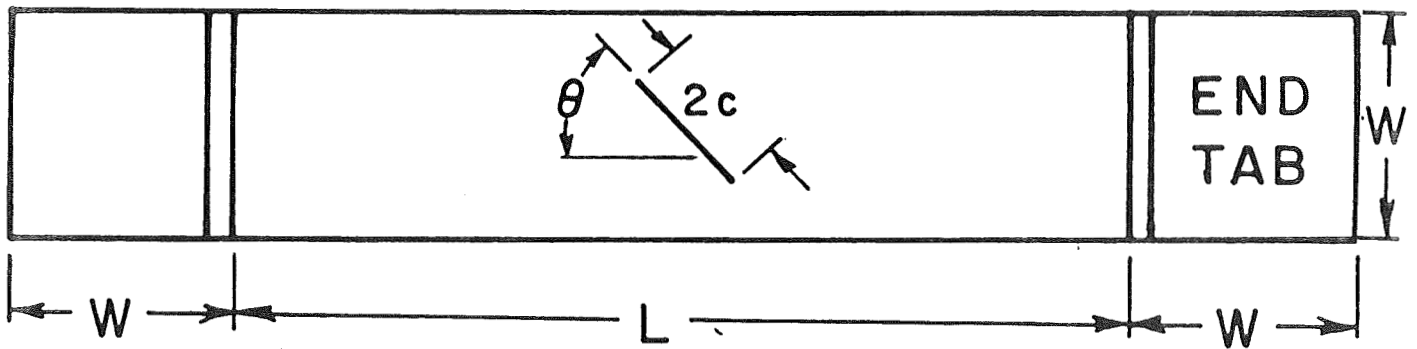
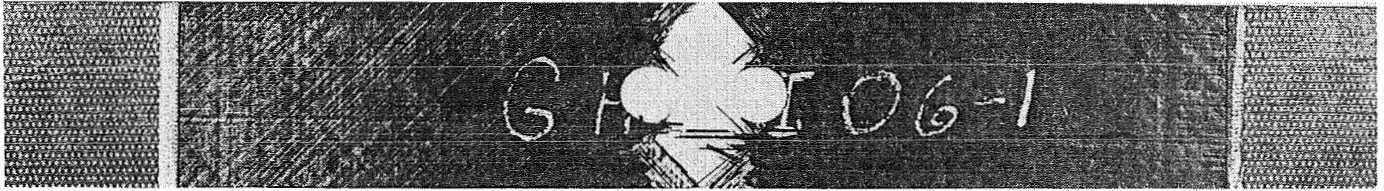
## AVERAGE STRESS CRITERION

$$K_Q = \sigma_0 \sqrt{\frac{\pi c (1 - \xi_4)}{(1 + \xi_4)}}$$

$$c \rightarrow \infty, K_Q \rightarrow \sigma_0 \sqrt{\frac{\pi d_0}{2}}$$







#### MODIFIED THEORY

$$d_0 = R^m / R_0^{m-1} C$$

R = HOLE RADIUS

$R_0$  = NORMALIZING FACTOR = UNIT LENGTH

m = EXPONENTIAL PARAMETER

C = NOTCH SENSITIVITY FACTOR

$$\frac{\sigma_N}{\sigma_0} = \frac{2}{[ (2 + \lambda^2 + 3\lambda^4) - (K_T^\infty - 3)(5\lambda^6 - 7\lambda^8) ]}$$

$$\lambda = \frac{C}{\left[ C + \left( \frac{R}{R_0} \right)^{m-1} \right]}$$

CASE 1:  $m = 0$

$d_0 = \text{CONSTANT}$ : ORIGINAL THEORY RECOVERED

CASE 2:  $0 < m < 1$

$$R \longrightarrow \infty, \lambda \longrightarrow 1, \frac{\sigma_N}{\sigma_0} \longrightarrow \frac{1}{K_T^\infty}$$

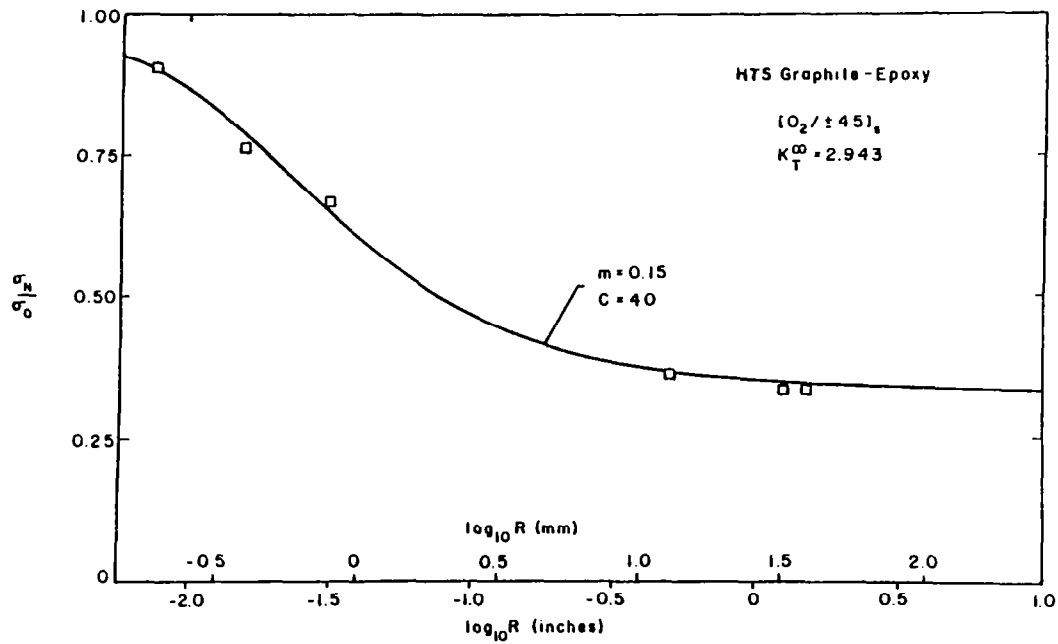
CASE 3:  $m = 1$

$$\lambda = \frac{C}{C+1}, \frac{\sigma_N}{\sigma_0} \text{ INDEPENDENT OF } R$$

CASE 4:  $m > 1$

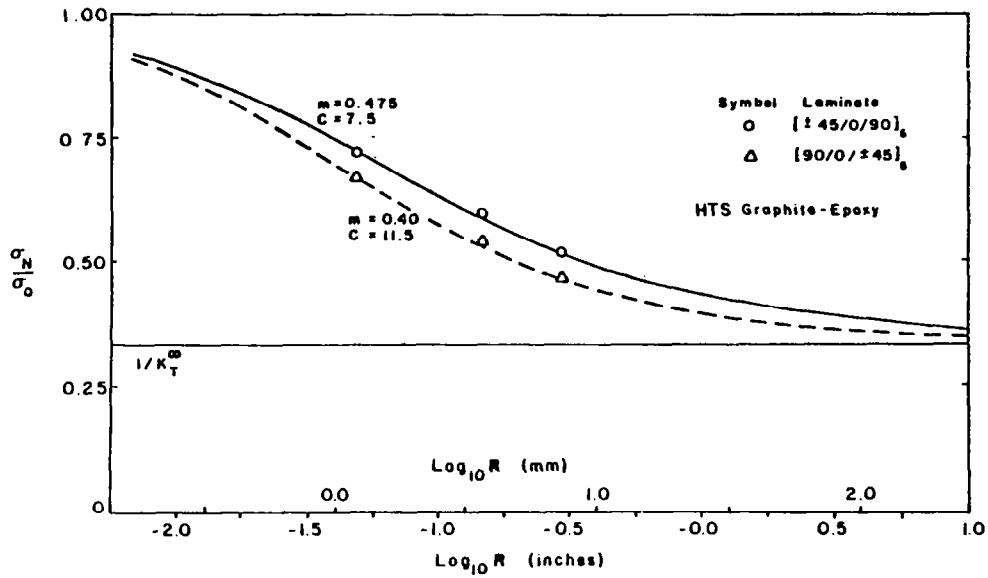
$$R \longrightarrow \infty, \lambda \longrightarrow 0, \frac{\sigma_N}{\sigma_0} \longrightarrow 1$$

$$0 \leq m < 1$$





# INFLUENCE OF STACKING SEQUENCE ON NOTCHED STRENGTH



## SUPERIMPOSING NOTCHED STRENGTH

### MATERIAL 1 AND MATERIAL 2

$$\left(\frac{\sigma_N}{\sigma_0}\right)_1 = \left(\frac{\sigma_N}{\sigma_0}\right)_2 \quad \text{IF } \lambda_2 = \lambda_1 \quad \text{AND } (K_T^{\infty})_2 = (K_T^{\infty})_1$$

$R$  = ARBITRARY RADIUS FOR MATERIAL 1

$R_e$  = RADIUS IN MATERIAL 2 PRODUCING SAME PERCENT STRENGTH REDUCTION

$$\frac{1}{C_1} \left(\frac{R_e}{R_0}\right)^{m_1-1} = \frac{1}{C_2} \left(\frac{R}{R_0}\right)^{m_2-1}$$

$$\text{LET } R_e = a_{cm} R$$

$$a_{cm} = \left(\frac{C_2}{C_1}\right)^{1/(m_2-1)} \left(\frac{R}{R_0}\right)^{\left(\frac{m_1-m_2}{m_2-1}\right)}$$

FOR CONSTANT m:

$$a_c = \left( \frac{C_1}{C_2} \right)^{1/(1-m)}$$

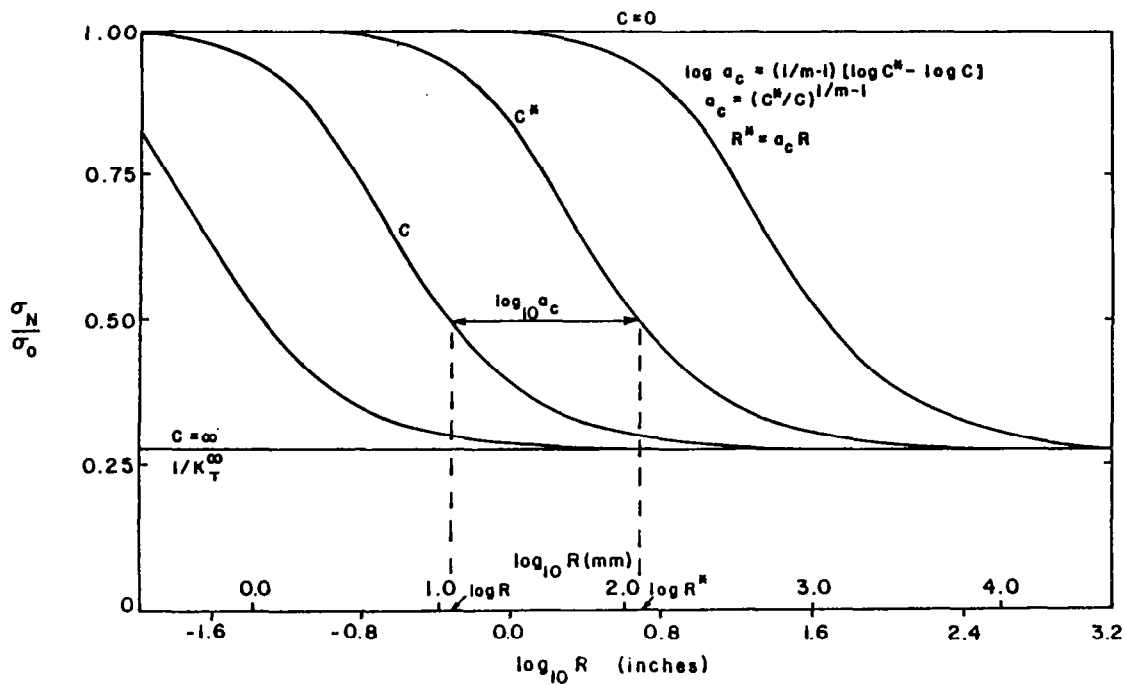
$$\text{LOG } a_c = \frac{1}{(1-m)} (\text{LOG } C_1 - \text{LOG } C_2)$$

FOR MOST COMPOSITES:

$$2 \leq K_T^\infty < 4$$

$$\frac{\sigma_N}{\sigma_0} \text{ INSENSITIVE FOR } R \leq 1.0$$

#### RADIUS-NOTCH SENSITIVITY FACTOR SUPERPOSITION



# DEFINE EXPONENTIAL SHIFT PARAMETER

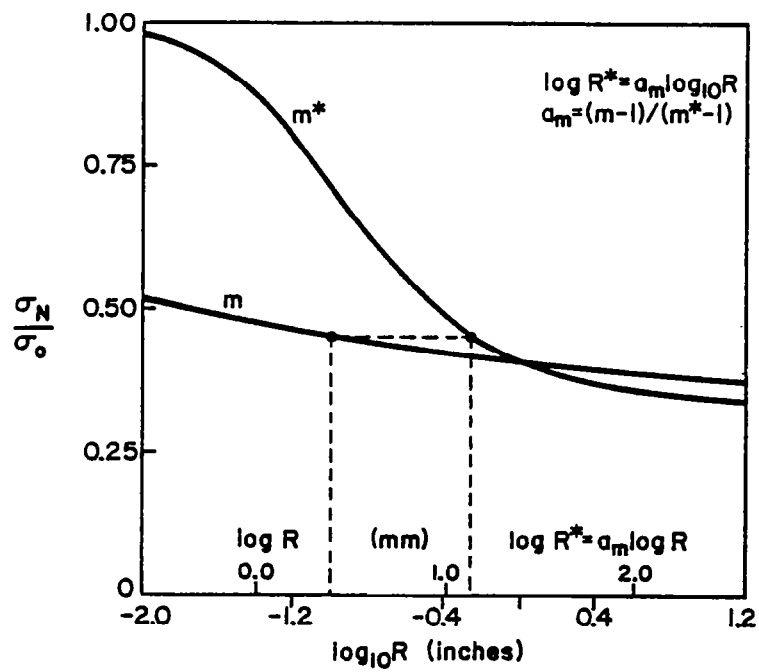
$$(K_T^\infty)_2 = (K_T^\infty)_1 ; C_2 = C_1$$

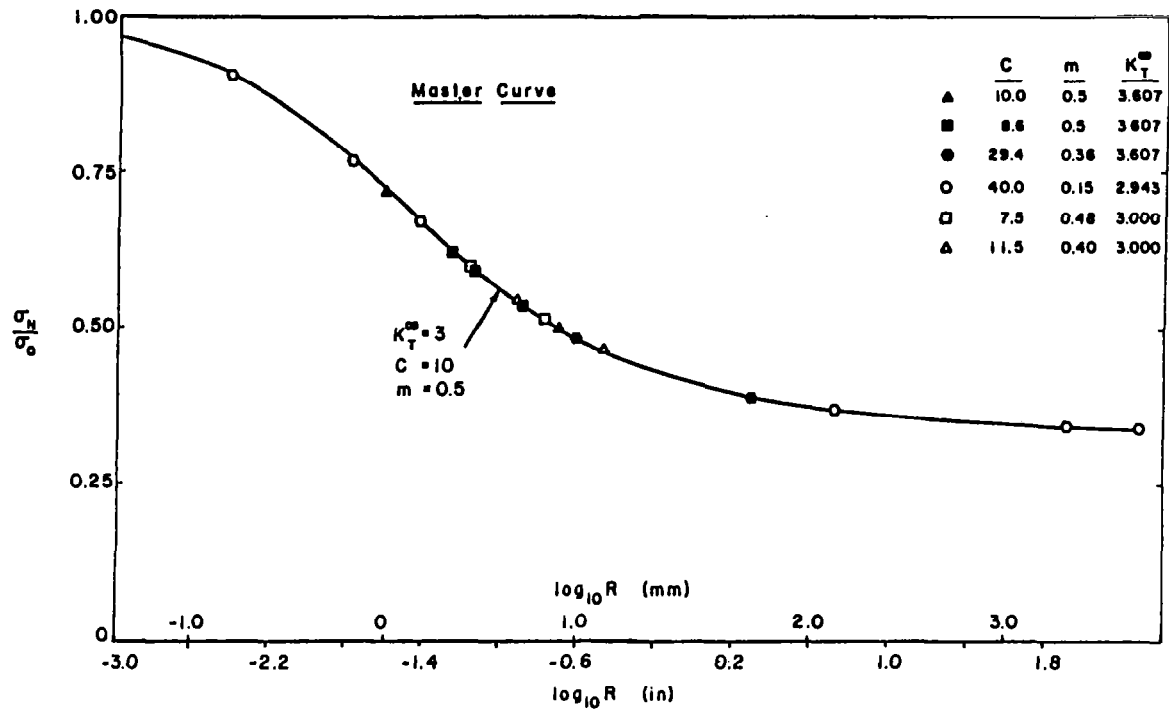
$$\frac{R_e}{R_0} = \left( \frac{R}{R_0} \right)^{a_m}$$

$$\text{LOG } R_e = a_m \text{ LOG } R$$

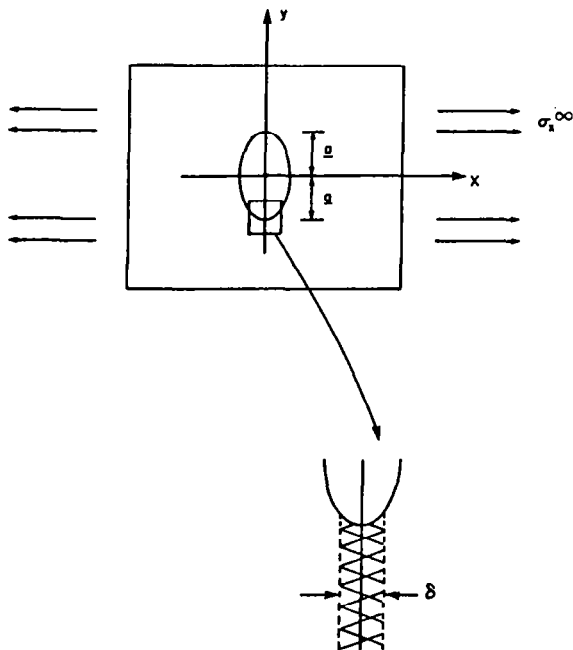
$$a_m = \left( \frac{m_1 - 1}{m_2 - 1} \right)$$

## EXPONENTIAL SHIFT PARAMETER





#### COMPARISON BETWEEN WEIBULL THEORY AND MODIFIED THEORY



$$-\delta/2 \leq x \leq \delta/2$$

$$a \leq y \leq L$$

$$-h/2 \leq z \leq h/2$$

$$\frac{\sigma_N}{\sigma_0} = \left( \frac{L}{\int_a^L [t(0, y)]^\alpha dy} \right)^{1/\alpha}$$

$$t(0, y) = 1 + 1/2\rho^2 + 3/2\rho^4$$

$$- (K_T^m - 3)(5/2\rho^6 - 7/2\rho^8)$$

$$\text{where } \rho = R/y$$

$$\sigma_N/\sigma_0 = [1 + 1/2\xi^2 + 3/2\xi^4$$

$$- (K_T^m - 3)(5/2\xi^6 - 7/2\xi^8)]^{-1}$$

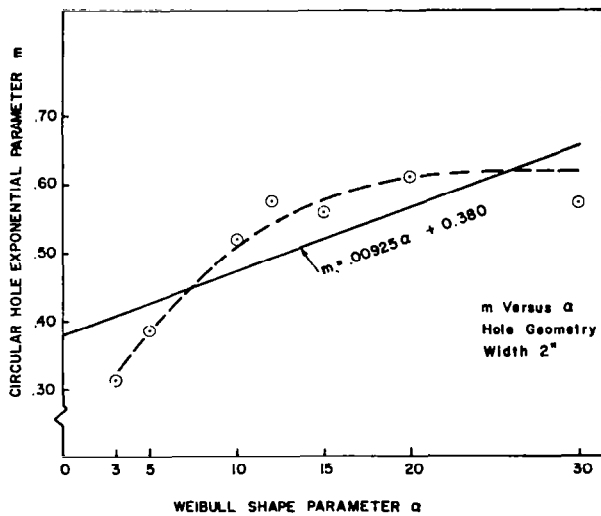
$$\text{where } \xi = R/(R + d_0) \text{ and } d_0 = R^m/K$$

Loading geometry - volume of numerical integration.

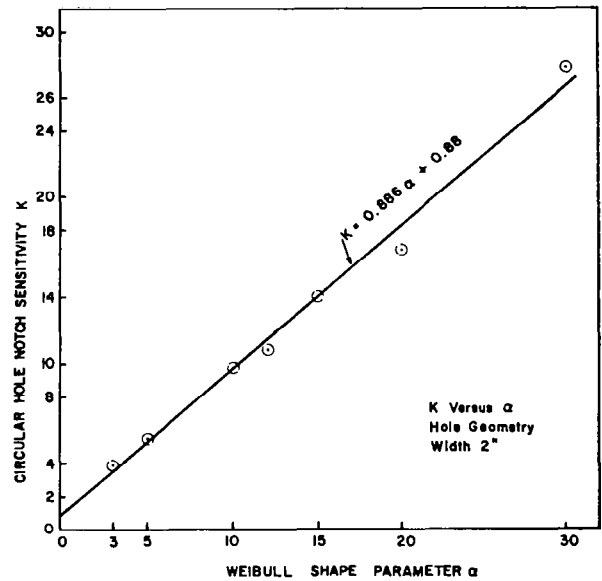
$$\ln d_0 = m \ln R - \ln K$$

Shape parameter $\alpha$	Exponential parameter $m$	Notch sensitivity $K$	Correlation coefficient* $r$
3	0.31	3.9	0.985
5	0.39	5.5	0.987
10	0.52	9.7	0.990
12	0.57	10.8	0.986
15	0.56	14.0	0.983
20	0.61	16.8	0.996
30	0.57	27.8	0.977

\* Correlation coefficient from the least squares fit of linear equation,  
 $\ln d_s = m \ln R - \ln K$ ; minimum three points (radii) used for each given  $\alpha$ .



Exponential parameter  $m$  as a function of shape parameter  $\alpha$ .



Notch sensitivity  $K$  as a function of shape parameter  $\alpha$ .

#### COMMENT

An interesting observation has been made (ref. 1) concerning the Whitney "point stress" or "average stress" criterion. In particular, the center-notched unidirectional laminate with no notch tip damage is shown to have a square root type stress distribution with an equivalent notch length which differs from the actual number of broken fibers by a small but constant amount, independent of the number of broken fibers. This is consistent with the assumptions leading to the "point stress" and "average stress" criterion. This is not the case if notch tip damage (in the form of matrix yielding or transverse broken fibers) is present, and it is shown that the damage produces stresses in the unbroken fibers which are less severe than a square root behavior, and also that an analogous equivalent notch length does not exist.

#### REFERENCE

1. Goree, J. G.; Dharani, L. R.; and Jones, W. F.: Mathematical Modeling of Damage in Unidirectional Laminates. NASA CR-3453, 1981.

James G. Goree  
Clemson University

#### COMMENT

The Weibull distribution function is based on a weakest link concept of failure, which in general is not applicable to composites. This concept underlies the size effect which would account for the difference between bending and uniform tensile strength in purely brittle materials. The difference between tensile and bending strengths in composites is more likely related to the "gradient" effect, which allows localized failure and stress redistribution, as in the case of elastic-plastic materials.

I. M. Daniel  
Illinois Institute of Technology

## COMMENT

The present point stress and average stress criteria relate the notched strength of a laminate to the average strength of a relatively long tensile coupon. Tests of notched specimens in which microstrain gages have been placed at or near the edges of the holes have measured strains much larger than those measured in an unnotched tensile coupon. Furthermore, orthotropic stress concentration analyses of failed notched laminates have also indicated that failure occurred at strains much larger than those experienced on tensile coupons with normal gage lengths. Earlier, both Hahn (ref. 1) and Wu (ref. 2) presented data that related fiber strength to gage length. This suggests that the high strains at the edge of a hole can be related to the very short length of fiber subjected to these strains. Since the length of fiber that is highly strained is proportional to the hole size, this would explain the higher strains to failure measured at the edge of smaller holes.

Lockheed has attempted to correlate a series of tests of several laminates with holes ranging from 0.19 to 0.50 in. Although the average stress criterion correlated well with test results for hole sizes equal to or greater than 0.50 in., it overestimated the laminate strength in the range of hole sizes from 0.19 to 0.38 in. It thus appears that we need a theory that is based on the mechanics of failure and is more generally applicable to the range of hole sizes and the varieties of laminates found in aircraft construction.

## REFERENCES

1. Hahn, H. T.: Failure Mechanisms. Failure Analysis and Mechanisms of Failure of Fibrous Composite Structures, NASA CP-2278, August 1983, pp. 153-171.
2. Wu, Edward M.: Strength Theories of Composites: Status and Issues. Failure Analysis and Mechanisms of Failure of Fibrous Composite Structures, NASA CP-2278, August 1983, pp. 173-189.

Larry Fogg  
Lockheed-California Company